Granular temperature: Experimental analysis

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The concept of granular temperature, introduced in the kinetic theory of rapid granular flow, is discussed here from experiments on a large air blowing table. Essentially, because granular systems are dissipative, the "temperature" is dependent on particle mass. This leads to a lack of equipartition. Then, granular temperature cannot be defined.

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Many authors working on granular media have tried to use the main concepts of thermodynamics in order to explain the behavior of these systems. For example, in the 1970s many papers were devoted to the question of the entropy of powders. More recently, Edwards, and coworkers [1,2] have built a thermodynamic "theory of powders" in which the internal energy of classical thermodynamics is replaced by the volume V, and the equivalent of the temperature is the compactivity $\chi=\partial V/\partial S$, where S is the entropy. A more classical point of view is described by Herrmann [3], who proposed another thermodynamic approach to granular media in which the granular temperature is defined. This temperature was introduced explicitly by Jenkins and Savage [4] and Haff [5] in their efforts to propose a theoretical description of rapid granular flows based on the kinetic theory of gases. The analogy is clear between molecular chaos, on which kinetic theory is based, and the disordered motion of grains in a strongly vibrating box [6,7] or at the surface of an inclined chute [8,9]. This kinetic theory of rapid granular flows has led to some important progress in this field in the last 10 years (for reviews, see [10,11]).

By definition, the kinetic temperature of a gas is proportional to the mean kinetic energy of the molecules: it has sense only if on average the kinetic energy is the same for each molecule in the gas (or for each degree of freedom), i.e., the equipartition of energy is respected. And this is the central question if one wants to use the concept of granular temperature since a granular medium is generally a mixture of grains of different sizes, shapes, or physical properties in which energy is dissipated. The system can be stationary only if there is balance between energy dissipated and energy supplied by a reservoir (or gravity field in the case of grains moving down an inclined chute). Then the question is not only to understand how the energy is dissipated (and exchanged) during collisions, but also how it is gained *between* collisions.

Numerical simulations using molecular dynamics are a good tool to analyze the limits of such a theory, but the energy dissipation is in some sense artificial. In our laboratory, we have built an experimental system well adapted to study this kind of problem: a large blowing air table [12,13], on which disks or pentagons are moving freely, without solid friction with the table. This system was also used to study experimentally the limits of the kinetic model for granular media [14].

The air table has been previously described [12]. Essentially, this table was built to permit experimental studies on the statistics of many two-dimensional (2D) systems, paying particular attention to the geometry of packing of disks, ellipses, or pentagons and mixtures of disks [15–17]. In this geometrical context, this tool is very comparable to the one built by Pouligny *et al.* [18]. Our air table (Fig. 1) consists of a horizontal porous slab made of sintered bronze of 50×50 cm^2 area and 0.5 cm thickness above a reservoir of pressurized air (at the exit of a long vertical wind channel). This air flow through the plate can be controlled by adjusting a rheostat connected to two fans at the bottom of the wind tunnel. The fan power is directly related to the average total pressure of air above the table surface [12]. Let us remark that the geometrical characteristics of the wind channel are chosen in order to obtain a laminar and homogeneous flow at the entrance of the porous medium. Light flat

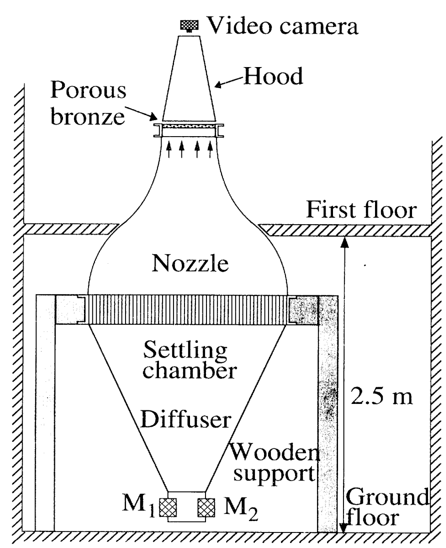

FIG. 1. Air flow device.

disks placed on the table float on a cushion of air with a practically negligible viscous friction. A special pyramid-shaped hood is placed above the air table, to eliminate boundary layer effects (except collisions with the wall). The gas particles are constituted of small disks (styrene) 1 mm thick and of diameter 8 mm for a monosize granular medium and for a bidisperse granular medium with diameters of 8 and 10 mm. These disks are sustained on the work surface, and move in a random motion on the air table. Such motion is due to two reasons. The first one is the existence of local fluctuations in the air velocity field and the other one is related to the existence of geometrical defects in the disks (they are not perfectly flat) or also, and perhaps essentially, to disequilibria created during collisions. Between collisions, the particle can be considered as uniformly accelerated, as shown experimentally [14], with an acceleration which does not depend on the surface packing fraction. Our gas is then a dissipative system (energy is essentially dissipated by shocks) connected to a reservoir of energy, the air flow system. Finally, a video system allows us to record the dynamical behavior of the granular media and then to analyze the image sequences.

In order to test the validity of the concept of temperature on the table, we have performed experiments in a dilute gas (surface fraction C between 0.6% and 5.5%) and a "dense gas" (surface fraction between 5.5% and 22%) formed by systems of equal disks and at a given exit air pressure $P=175$ Pa. As illustration, the mean free path λ (i.e., the length over which the particle is accelerated or the mean displacement of a particle between two collisions) for a surface fraction of 5.5% (i.e., with roughly 275 disks on the table) is of the order of 10 times the particle radius: the system remains relatively dilute, even if the size of the grains is not negligible compared to λ. Let us add that, for the range of surface fraction used, disk-disk hydrodynamical effects can be neglected: roughly, the effective radius R_{eff} of a disk of radius R, taking into account the hydrodynamic interaction, is $R_{\text{eff}}=1.07R$ [13]. It is not possible to determine correctly the distribution of velocities for packing fractions larger than 22%, because λ becomes too small compared to R (i.e., of the same order).

The first thing to do is to determine the velocity distribution corresponding to our "gas." In practice, we record the images corresponding to the particle motions of the gas constituted by equal disks at a given surface fraction and at a given exit air pressure flow during 30 min. After that and by using a TV screen, we choose 10 disks and we obtain their local velocity by measuring their successive positions at time intervals of 0.2 sec (five image frames of $\frac{1}{25}$ sec). The typical experimental error on each local velocity is estimated to be of the order of 15%. In such a way, a statistics with about 300 measurements for each surface fraction C allows us to obtain the corresponding experimental velocity distribution. Let us remark that we have eliminated all those particles that have suffered a collision (with a particle or with the wall) during measurements. Figure 2 shows the experimental velocity distribution for a gas of equal disks and the corresponding Maxwell-Boltzmann velocity distribution, in

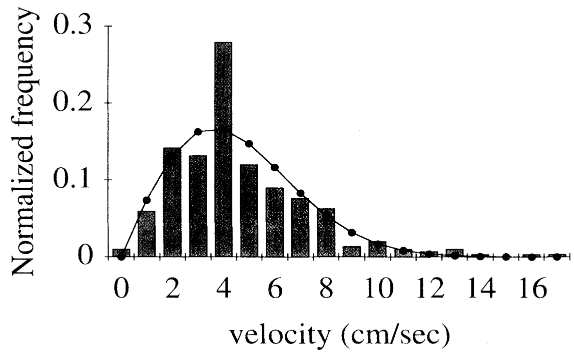

FIG. 2. Comparison between Maxwell-Boltzmann velocity distribution (–·–) and experimental velocity distribution (gray) for gas particles of a surface fraction of 2.5%.

which the only introduced fitting parameter is the mean experimental square velocity $\langle v^2 \rangle$. We observe that the agreement between the two distributions is not so bad. In the experimental distribution, because of our experimental technique, the probability for a particle to have a small velocity (i.e., essentially a particle which just has had a shock) is underestimated. It is also the case for large velocities when the influence of the finite size of the working surface cannot be neglected any longer: some particles for which the distance between two successive shocks is large are not taken into account because they can reach the table limits. Overestimation of the frequency of the velocities near the mean one is observed whatever the surface packing fraction. When C increases, the width of the distribution decreases (as shown Fig. 3), as the mean free path λ decreases. Nevertheless, we can assume, in first approximation, that our experimental velocity distribution can be described by a Maxwell-Boltzmann distribution. Then we estimate the experimental granular temperature T^* as being

$$T^* = \frac{m}{k}\frac{\langle v^2 \rangle}{2}, \qquad (1)$$

where $\langle v^2 \rangle$ is the mean square velocity. In molecular kinetic theory, m is the mass of the molecule and k the

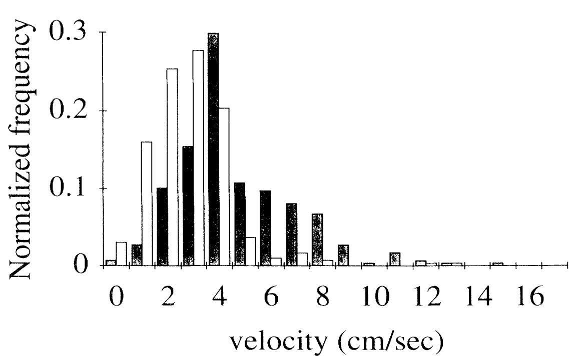

FIG. 3. Experimental velocity distribution for a monosize packing at a surface fraction $C=1\%$ (gray) and $C=22\%$ (white).

Boltzmann constant, introduced to define the kinetic energy of one molecule for one degree of freedom. In the case of granular media, such a definition is meaningless, because the mass is very large. So a temperature scale has been defined for the granular temperature by assuming that the ratio $\alpha = m/k = 1$ [4]. But this simplification leads to a temperature normalized by the mass of the grain, which is meaningless if the system is a mixture of grains of different masses (this will be discussed in more detail below).

In perfect gases, without external input of energy, the temperature remains constant (elastic collisions) and it is independent of the gas concentration. We have performed some experiments in order to study the evolution of the pseudotemperature T^* on the air table with the surface fraction C from 0.6% to 22% at a given air pressure flow. Figure 4 gives the experimental variation of T^* with the surface fraction C. The experimental error in the temperature values was estimated to be of the order of $\pm 5\%$. We observe that T^* decreases when the surface fraction C increases [14]. In fact, T^* is determined by the balance between the energy loss by the system (essentially during the collisions) and the energy gained by the system in exchange with the reservoir (between consecutive collisions). The mean square velocity is proportional to the mean free path λ, which is inversely proportional to the surface fraction C. S_0, the global loss of energy, is proportional to the number of collisions (i.e., to C), and to the—average—energy loss during the collisions. It is not possible, in our case, to evaluate this last term (in particular, the influence of the fluctuating nonflatness of the disks) but a strong decrease of T^* with C was foreseeable.

In a mixture of perfect gases at equilibrium, equipartition is fulfilled and all the particles have the same temperature: the temperature of the system. Molecular chaos (and elastic shocks) leads to a perfect exchange of energy between different species. The study of the granular temperature of the equal disks system introduced above does not allow us to decide if this term temperature has a meaning. So, the real question is whether different species in a mixture of grains are at the same temperature. In order to study that, we have done two different experiments.

(i) In the first one, we have measured separately the temperature of two systems of equal disks, with diameters 8 and 10 mm, respectively, at the same surface fraction $C = 2.5\%$.

(ii) In the second one, we have studied the velocity distribution in a mixture of small (8 mm diameter) and large (10 mm diameter) disks with the same percentage area, the corresponding total surface fraction being also $C = 2.5\%$. For this value of C, the diameter of the particle remains small compared to the mean free path λ. Let us add that all the experiments have been performed at the same air pressure flow, $P = 175$ Pa.

What are the consequences of the size of a disk on its dynamics on the table? The α term in the granular temperature definition [Eq. (1)] must be modified here, in order to take into account the fact that only the kinetic energy is exchanged. So we must reintroduce the mass in the definition of the temperature:

$$T^* = \mu \frac{\langle v^2 \rangle}{2} , \qquad (2)$$

where μ is here the mass of the considered disk divided by the mass of an 8 mm disk.

In systems with disks of the same thickness, but different diameters, the mean square velocity is independent of the mass (or of the disk size), but the temperature is roughly proportional to the mass (i.e., surface) of the disks. In the first series of experiments, we have obtained temperature values of $T_8^* = 12.6$ and $T_{10}^* = 19.1$ (T_i^* is the temperature corresponding to diameters of $i = 8$ and 10 mm). This difference is relatively large, and appreciably decrease for mixtures. In this last case, the temperature for a particle of size 8 mm is $T_8'^* = 13.6$, and that of a particle of size 10 mm is $T_{10}'^* = 15.5$. The shocks have led to a redistribution of energy: the temperature of small disks is larger in the mixture than in the "pure" system, and the large disk temperature is smaller. Note that for each species the experimental velocity distribution is not far from being a Maxwellian one, as we can see in Fig. 5. The same results have been found in experiments performed at a surface fraction $C = 5.5\%$.

We can conclude that the exchange of energy in shocks is clearly not sufficient to equalize the distributed energy.

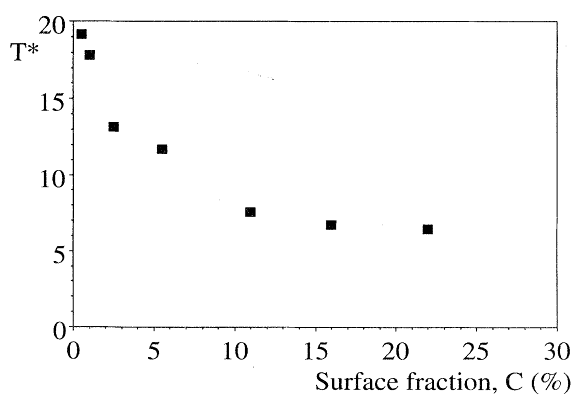

FIG. 4. Pseudotemperature T^* variation as a function of the surface fraction C.

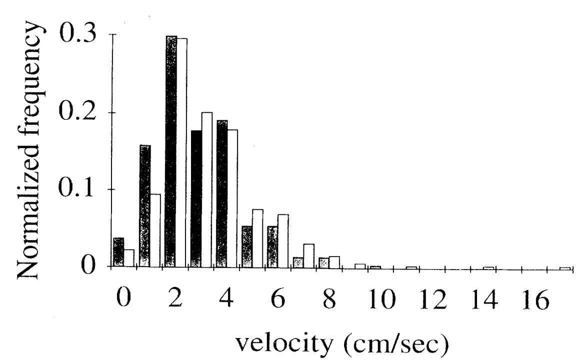

FIG. 5. Experimental velocity distribution for each species of a mixture of grains of (white) 8 mm diameter and (gray) of 10 mm diameter for a total surface fraction C of 2.5%.

This lack of equipartition is due to the fact that the energy exchange between a given particle and the reservoir is dependent on the mass of this particle. Note that this lack of equipartition can also be due to a difference in the energy loss during collisions related to the disk sizes (friction, rotation, etc.). So a granular temperature cannot be defined in this case (one can think that this lack becomes less important for large concentrations of particles, because the time during which energy is gained becomes smaller and smaller). Let us say that this lack of equipartition is generally observed in flowing granular media: A distribution of particle masses, shapes, or physical properties is always observed, and leads to a difference in the energy exchange of each species, which is not sufficiently redistributed during collisions.

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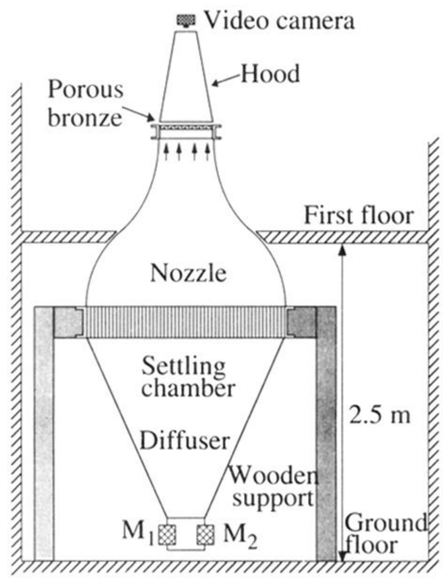

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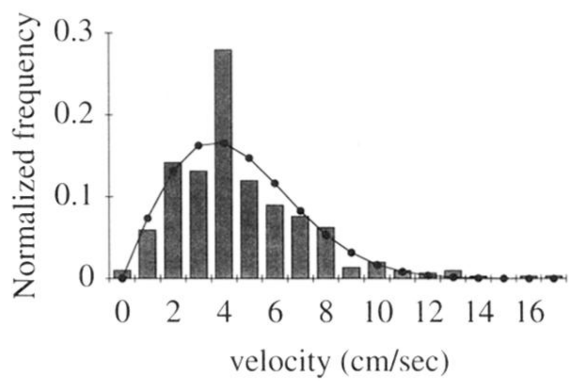

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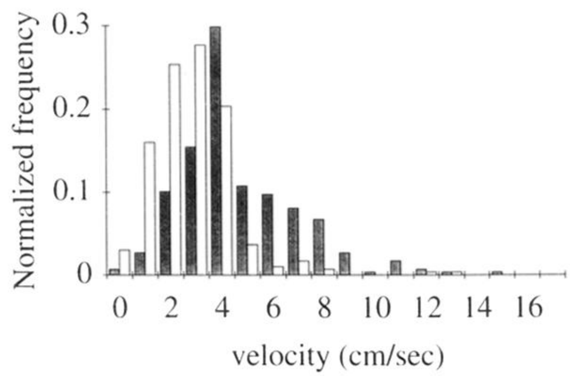

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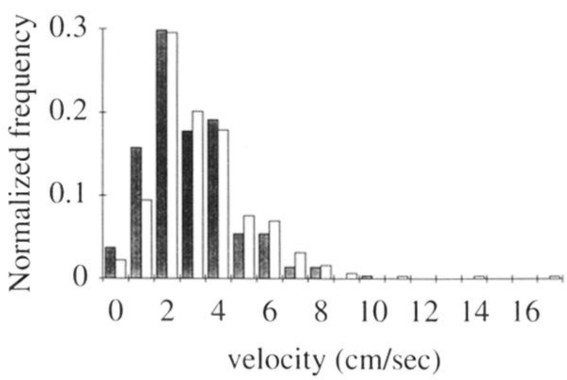

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